

Dependability Analysis of Heterogeneous Distributed Systems

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Abstract—Distributed computing is an enabling technology for many current and future application areas and is increasingly pervasive. Distributed systems consisting of interconnected sets of dissimilar hardware or software systems are commonly referred to as heterogeneous distributed systems. As a consequence of large scale, heterogeneity is often inevitable in distributed systems. Furthermore, often heterogeneity is preferred by many users because heterogeneous distributed systems provide the flexibility to their users of different computer platforms for different applications. A Markov model of the Heterogeneous distributed system (HDS) with a circular consecutive $k - out - of - n : F$ configuration is developed and the availability and reliability measures are obtained.

Index Terms— Heterogeneous Distributed System; Markov Model; Availability; Reliability.

1 INTRODUCTION

Fault tolerance in distributed systems is strictly related to their property of dependability and can be implemented in two ways: at the architectural level and at the application level [1]. Distributed systems have been widely used in many critical systems such as banking systems, military systems, nuclear systems, aircraft systems, power systems and so forth [2], [3].

A heterogeneous distributed system (HDS) due to its diversity is more difficult to design than homogeneous distributed systems in which each system is based on the same, or closely related, hardware and software. However, as a consequence of large scale, heterogeneity is often inevitable in distributed systems. Furthermore, often heterogeneity is preferred by many users because HDS's provide the flexibility to their users of different computer platforms for different applications. For example, a user may have the flexibility of choosing a super-computer for simulations, a Macintosh for document processing, and a UNIX workstation for program development.

When the distributed system is homogeneous, the problem is simplified mathematically, but in real world systems like Internet, the computers have heterogeneous characteristics. An upper bound of the reliability of heterogeneous distributed system was obtained by the construction of a tripartite graph [4]. The service reliability and service availability of centralized heterogeneous distributed systems as an important measure of quality of service was studied by means of Markov model [5]. The capacity-based fuzzy reliability and perfor-

mance-based fuzzy reliability of large distributed non-homogeneous networks was obtained by means of capacity and performance based models [6].

Task allocation for maximizing reliability of heterogeneous distributed systems was obtained using a simulated annealing approach [7]. First an allocation model was developed by them for reliability based on cost function representing the unreliability caused by the execution of tasks on the system processors and the unreliability caused by the inter-processor communication time subject to constraints imposed by both the application and the system resources. Next a heuristic algorithm is derived from the simulated annealing technique to quickly solve the task allocation problem. Further, they compare the quality of the solutions with the branch and bound technique.

A dynamic and reliability-driven real-time fault tolerant scheduling algorithm was proposed for heterogeneous distributed systems [8]. Primary-backup copy scheme is leveraged in to the algorithm to tolerate both hardware and software failures. Their algorithm's main objective is to dynamically schedule independent, non-preemptive aperiodic tasks so that reliability is enhanced without additional hardware costs. Their algorithm is superior to the existing scheduling schemes in the literature due to its flexibility and the backup copies for active and passive forms. High availability was achieved based on residual lifetime analysis for heterogeneous distributed computational systems [9]. An availability model was provided taking into account system's expected residual lifetime. Further an objective function was proposed about the model and a heuristic scheduling algorithm to maximize the availability with the make span constraint.

There are several models such as Markov, graph and fuzzy available in the literature for HDS reliability and availa-

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bility analysis. Most distributed systems follow regular topologies to interconnect the computing nodes such as circle, rectangular mesh, hypercube and star, simply because these structures allow simpler routing algorithms. The HDS consists of homogeneous distributed systems (or nodes) connected by gateways (GW) as shown in Figure 1. The nodes of a HDS are connected in a ring topology as the ring topology allows simpler routing algorithms, higher fault tolerance ability and reliability [10], [11]. The communication links and gateways are assumed to be perfect and a node failure does not partition the network. The HDS is a circular consecutive $k-out-of-n:F$ system with non-identical nodes as shown in Figure 2. The system fails whenever k consecutive nodes are failed, where $k \leq n$.

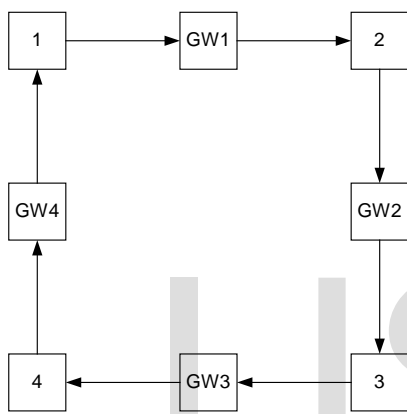


Fig. 1. Heterogeneous Distributed System

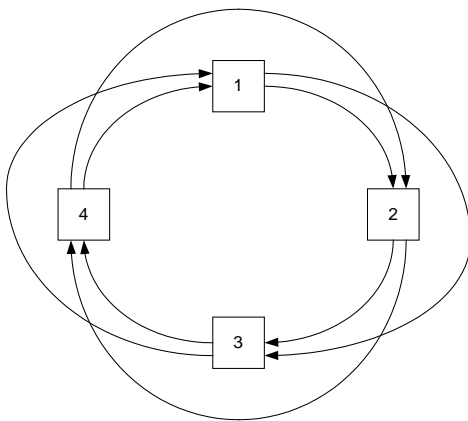


Fig. 2. Circular Consecutive 2-out-of-4:F system

A consecutive $k-out-of-n:F$ system consists of n components C_1, C_2, \dots, C_n which either fail or operate. The system fails whenever k consecutive components are failed, where $k \leq n$. If C_1, C_2, \dots, C_n are arranged in a line, then the system is linear. If they form a circle, then the system is circular. For circular systems, it is assumed the components are

labeled clockwise from 1 to n . The consecutive systems with identical components are analyzed by several researchers as shown in [12]. However, the analysis of consecutive systems with non-identical components is rare and only a few researchers have made contributions [13]-[16]. The consecutive $k-out-of-n:F$ system models are useful for design of integrated circuits, microwave relay stations in telecommunications, oil pipe systems, vacuum systems in accelerators, computer ring networks and spacecraft relay stations.

The rest of the paper is organized as follow: Section 2 presents a Markov model of the HDS. In section 3, we present the numerical results of a four-node HDS. We perform a sensitivity analysis of system availability and system reliability on various parameters. In section 4, we give our conclusions.

1.1 Notation

n	number of nodes in HDS
k	least number of consecutive nodes in failed state
λ_i	failure rate of i^{th} node
μ	repair rate of node
$P_i(t)$	probability of HDS in state i at time t
$A(t)$	availability function
$R(t)$	reliability function

2 MODEL DESCRIPTION

In this section a Markov model of HDS is described based on several assumptions.

1. The heterogeneous distributed system has a circular consecutive $k-out-of-n:F$ configuration as this configuration permits higher fault tolerance ability and reliability.
2. The HDS consists of n working homogeneous distributed systems connected by gateways.
3. The communication links and gateways are assumed to be perfect.
4. Each node (homogeneous distributed system) and HDS has only two states: up (working state) and down (failure state).
5. The nodes are mutually independent.
6. The failure time of the node i is exponentially distributed with parameter λ_i .
7. There is a single repair facility to repair the nodes. When a node fails, repair immediately commences. The repair is carried out in a first come first serve (FCFS) basis. The repair time of a node is independent and identically distributed (i.i.d) following exponentially distribution with parameter μ .

8. The HDS fails whenever at least k consecutive nodes fail where $k \leq n$.
9. The repaired unit is as good as new and immediately shares the load of the system.
10. The probability that two or more nodes are restored to the working condition or become failed in a small time interval is negligible.

The state transition diagram of the four-node HDS is shown in Figure 3. A description of the states of HDS is given in Table 1. The states 0,1,2,3,4,6,10,11 and 15 in Figure 3 are the up states and the remaining states are down states of the system. The failure rate of i^{th} node is $\lambda_i, i = 1,2,3,4$.

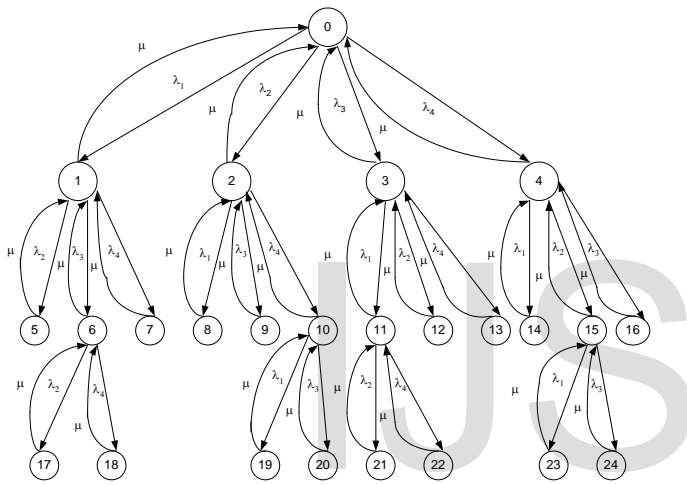


Fig. 3. State transition diagram of 4-node HDS

TABLE 1: STATE DESCRIPTION

State	Description
0	All four nodes of the system are working (initial state of the system)
1	Node 1 is in a failed state and nodes 2,3,4 are working
2	Node 2 is in a failed state and nodes 1,3,4 are working
3	Node 3 is in a failed state and nodes 1,2,4 are working
4	Node 4 is in a failed state and nodes 1,2,3 re working
5	Nodes 1,2 are in failed state and nodes 3,4 are working
6	Nodes 1,3 are in failed state and nodes 2,4 are working
7	Nodes 1,4 are in failed state and nodes 2,3 are working
8	Nodes 2,1 are in failed state and nodes 3,4 are working

9	Nodes 2,3 are in failed state and nodes 1,4 are working
10	Nodes 2,4 are in failed state and nodes 1,3 are working
11	Nodes 3,1 are in failed state and nodes 2,4 are working
12	Nodes 3,2 are in failed state and nodes 1,4 are working
13	Nodes 3,4 are in failed state and nodes 1,2 are working
14	Nodes 4,1 are in failed state and nodes 2,3 are working
15	Nodes 4,2 are in failed state and nodes 1,3 are working
16	Nodes 4,3 are in failed state and nodes 1,2 are working
17	Nodes 1,3,2 are in a failed state and node 4 is working
18	Nodes 1,3,4 are in failed state and node 2 is working
19	Nodes 2,4,1 are in failed state and node 3 is working
20	Nodes 2,4,3 are in failed state and node 1 is working
21	Nodes 3,1,2 are in failed state and node 4 is working
22	Nodes 3,1,4 are in failed state and node 2 is working
23	Nodes 4,2,1 are in failed state and node 3 is working
24	Nodes 4,2,3 are in failed state and node 1 is working

The corresponding set of Kolomogorov's differential equations is:

$$\frac{dP_0(t)}{dt} = -(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)P_0(t) + \mu P_1(t) + \mu P_2(t) + \mu P_3(t) + \mu P_4(t) \quad (1)$$

$$\frac{dP_1(t)}{dt} = -(\mu + \lambda_2 + \lambda_3 + \lambda_4)P_1(t) + \lambda_1 P_0(t) + \mu P_5(t) + \mu P_6(t) + \mu P_7(t) \quad (2)$$

$$\frac{dP_2(t)}{dt} = -(\mu + \lambda_1 + \lambda_3 + \lambda_4)P_2(t) + \lambda_2 P_0(t) + \mu P_8(t) + \mu P_9(t) + \mu P_{10}(t) \quad (3)$$

$$\frac{dP_3(t)}{dt} = -(\mu + \lambda_1 + \lambda_2 + \lambda_4)P_3(t) + \lambda_3 P_0(t) + \mu P_{11}(t) + \mu P_{12}(t) + \mu P_{13}(t) \quad (4)$$

$$\frac{dP_4(t)}{dt} = -(\mu + \lambda_1 + \lambda_2 + \lambda_3)P_4(t) + \lambda_4 P_0(t) + \mu P_{14}(t) + \mu P_{15}(t) + \mu P_{16}(t) \quad (5)$$

$$\frac{dP_5(t)}{dt} = -\mu P_5(t) + \lambda_2 P_1(t) \quad (6)$$

$$\frac{dP_6(t)}{dt} = -(\mu + \lambda_2 + \lambda_4)P_6(t) + \lambda_3 P_1(t) + \mu P_{17}(t) + \mu P_{18}(t) \quad (7)$$

$$\frac{dP_7(t)}{dt} = -\mu P_7(t) + \lambda_4 P_1(t) \quad (8)$$

$$\frac{dP_8(t)}{dt} = -\mu P_8(t) + \lambda_1 P_2(t) \quad (9)$$

$$\frac{dP_9(t)}{dt} = -\mu P_9(t) + \lambda_3 P_2(t) \quad (10)$$

$$\frac{dP_{10}(t)}{dt} = -(\mu + \lambda_1 + \lambda_3)P_{10}(t) + \lambda_4 P_2(t) + \mu P_{19}(t) + \mu P_{20}(t) \quad (11)$$

$$\frac{dP_{11}(t)}{dt} = -(\mu + \lambda_2 + \lambda_4)P_{11}(t) + \lambda_1 P_3(t) + \mu P_{21}(t) + \mu P_{22}(t) \quad (12)$$

$$\frac{dP_{12}(t)}{dt} = -\mu P_{12}(t) + \lambda_2 P_3(t) \quad (13)$$

$$\frac{dP_{13}(t)}{dt} = -\mu P_{13}(t) + \lambda_4 P_3(t) \quad (14)$$

$$\frac{dP_{14}(t)}{dt} = -\mu P_{14}(t) + \lambda_1 P_4(t) \quad (15)$$

$$\frac{dP_{15}(t)}{dt} = -(\mu + \lambda_1 + \lambda_3)P_{15}(t) + \lambda_2 P_4(t) + \mu P_{23}(t) + \mu P_{24}(t) \quad (16)$$

$$\frac{dP_{16}(t)}{dt} = -\mu P_{16}(t) + \lambda_3 P_4(t) \quad (17)$$

$$\frac{dP_{17}(t)}{dt} = -\mu P_{17}(t) + \lambda_2 P_6(t) \quad (18)$$

$$\frac{dP_{18}(t)}{dt} = -\mu P_{18}(t) + \lambda_4 P_6(t) \quad (19)$$

$$\frac{dP_{19}(t)}{dt} = -\mu P_{19}(t) + \lambda_1 P_{10}(t) \quad (20)$$

$$\frac{dP_{20}(t)}{dt} = -\mu P_{20}(t) + \lambda_3 P_{10}(t) \quad (21)$$

$$\frac{dP_{21}(t)}{dt} = -\mu P_{21}(t) + \lambda_2 P_{11}(t) \quad (22)$$

$$\frac{dP_{22}(t)}{dt} = -\mu P_{22}(t) + \lambda_4 P_{11}(t) \quad (23)$$

$$\frac{dP_{23}(t)}{dt} = -\mu P_{23}(t) + \lambda_1 P_{15}(t) \quad (24)$$

$$\frac{dP_{24}(t)}{dt} = -\mu P_{24}(t) + \lambda_3 P_{15}(t) \quad (25)$$

Initial conditions are $P_0(0) = 1$, and $P_i(0) = 0, i > 0$.

Taking Laplace transform of the differential equations (2.1) to (2.25) with initial conditions and by solving, the state probabilities are obtained. The closed form expressions of $P_i(t)$ are not shown due to the difficulty involved in Laplace inversion of functions by analytical method. Hence all state probabilities are obtained numerically using MATLAB. The availability, reliability and MTTF are calculated using the following formulas.

The availability of the system is calculated by

$$A(t) = \sum_i P_i(t),$$

where i is the working state of the system

The repair transition from each of the down states to up states is valid for the availability analysis and not valid for the reliability analysis of the HDS. i.e., the failure states are regarded as absorbing states.

The reliability of the system is calculated by

$$R(t) = \sum_i P_i(t),$$

where i is the working state of the system

The mean-time-to-failure of the system is calculated by

$$MTTF = \int_0^{\infty} R(t) dt.$$

3 NUMERICAL ILLUSTRATIONS

In this section, the dependability measures of HDS are presented. The illustrations are provided for a four node HDS.

3.1 Availability Measures

The HDS availability function is graphically represented in Figure 4 with repair rate as $\mu = 1$ per year. This figure shows the effect of node failure rates on HDS availability. It is observed that the availability increases with decrease in node failure rates and attains steady state after 2.5 years. The HDS availability function is graphically represented in Figure 5 with node repair rates as $\lambda_1 = 0.1, \lambda_2 = 0.2, \lambda_3 = 0.3, \lambda_4 = 0.4$ (per year). The effect of repair rate on HDS availability is shown in this figure. It is seen that the availability increases with increase in repair rate attains steady state after 2.5 years.

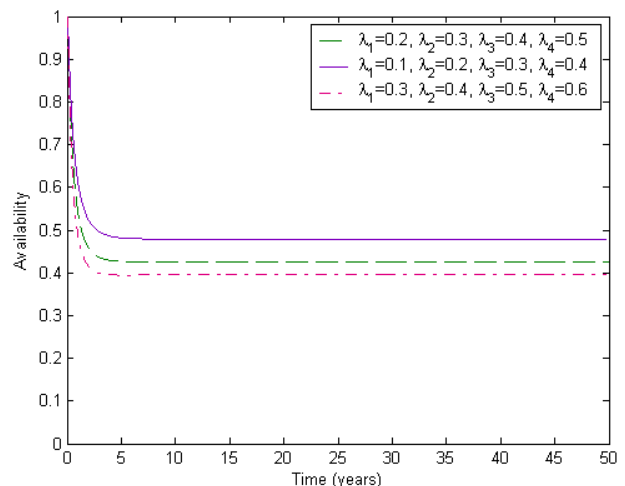


Fig. 4. Effect of Failure Rates on HDS Availability

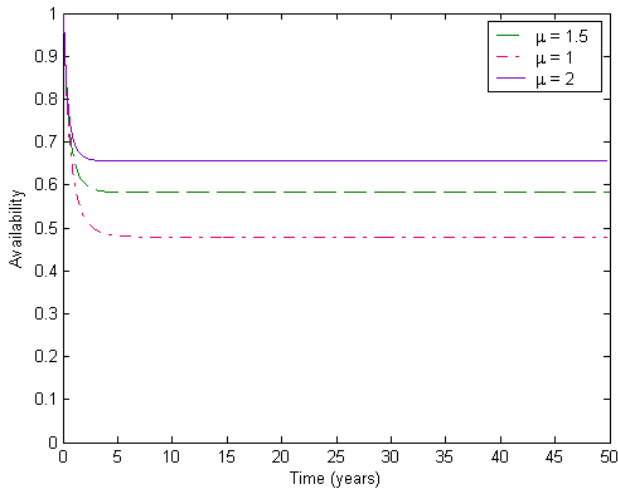


Fig. 5. Effect of Repair Rate on HDS Availability

3.2 Reliability Measures

HDS reliability function is depicted in Figure 6 with repair rate $\mu = 1$ per year. The effect of node failure rates on HDS reliability is shown in this figure. It is observed that the reliability increases with decrease in node failure rates. The HDS reliability function is depicted in Figure 7 with $\lambda_1 = 0.1$ per year, $\lambda_2 = 0.2$ per year, $\lambda_3 = 0.3$ per year, $\lambda_4 = 0.4$ per year. This figure shows the effect of repair rate on HDS reliability. It is observed that the reliability increases with increase in repair rate.

The effect of repair rate on HDS MTTF is shown in Figure 8. It is observed that MTTF increases with increase in repair rate and decreases with increase in node failure rates.

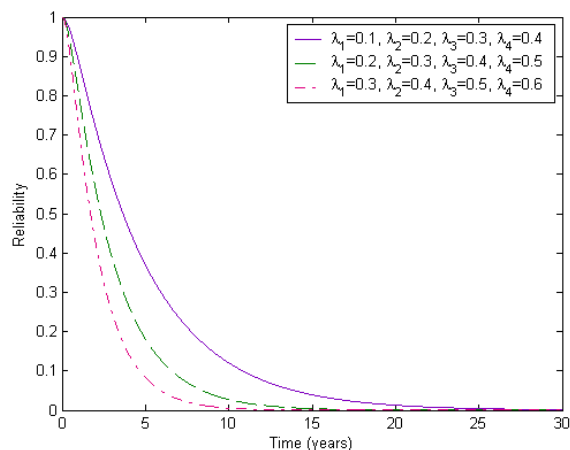


Fig. 6. Effect of Failure Rates on HDS Reliability

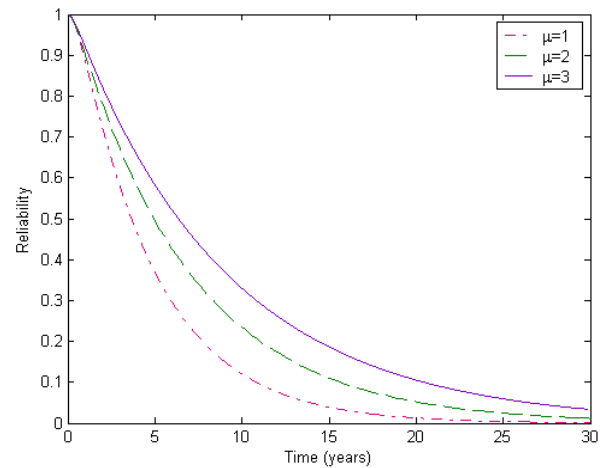


Fig. 7. Effect of Repair Rate on HDS Reliability

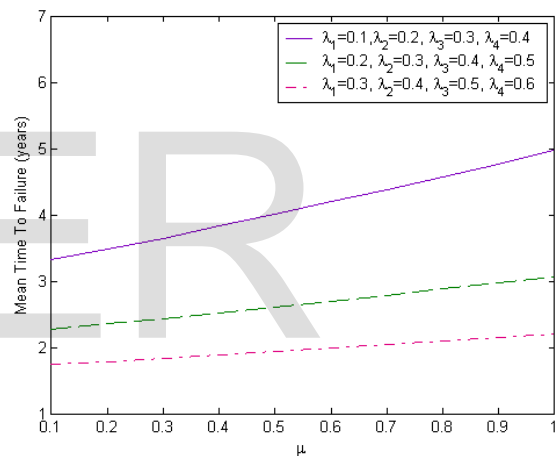


Fig. 8. Effect of Repair Rate on HDS MTTF

4 CONCLUSION

In this paper, a Markov model for the heterogeneous distributed system is described. The transient solutions of the system are obtained. Dependability measures such as availability, reliability and MTTF are calculated.

A particular case with $n = 4$ and $k = 2$ is analyzed numerically. The effect of node failure rates and node repair rate on HDS availability and HDS reliability is shown. It is observed that as node failure rates decrease and node repair rates increase, the HDS availability, HDS reliability and HDS MTTF increase. Although the analysis of a four node heterogeneous distributed system is discussed in this paper, the approach is applicable for a n -node heterogeneous distributed system. The developed model may be useful in computer ring networks and spacecraft relay stations.

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